

## Brookhurst Primary School

### Formal Written Calculations Policy – November 2006

#### **1 Aims and Purposes**

- 1.1 The purpose of the Formal Written Calculations Policy is to develop a consistent approach to the way in which pupils record Mathematical calculations. Through this, a clear progression in their recording will become evident.
- 1.2 It aims to inform teaching staff, non-teaching staff, parents and governors as to how the school have chosen to record written calculations. It is to supplement the existing Mathematics policy and to support medium and short term planning using the National Numeracy Strategy Renewed Framework document.
- 1.3 The policy will cover all Key Stages, showing where teaching input in the Foundation stage will be built upon as the child progresses through to the end of Key Stage 2. All four mathematical operations will be dealt with in turn, and worked examples will be included to clarify the methods to be adopted.

#### **2 Foundation Stage**

- 2.1 At the Foundation Stage, children are not required to carry out formal written calculations. They will be working towards the Early Learning Goals, learning to read and write numbers, understand and use the four operations and key vocabulary. Children's understanding of number is developed through practical activities. This understanding may be recorded through pictures and informal jottings.

We had 5 balloons, we lost 2.



*'Five take away two is three'.*  
 $5 - 3 = 2$  (modelled by teacher)

#### **3 Teaching and Learning**

- 3.1 Children need to be able to understand and use one formal written method for each operation. They should be working towards a method that helps them to calculate efficiently and one that can be understood by other people.
- 3.2 Some children will not progress through all the methods shown here.
- 3.3 Written methods should complement mental methods, but children should be able to choose the most efficient method for a problem, be it mental, written or using a calculator.
- 3.4 Children should be taught written methods so that they can represent practical work and so they can record and explain mental calculations. It also helps children to keep track of procedures in longer problems and can prepare them for calculations in algebra.
- 3.5 Support and guidance are needed through the important stages of written mathematics.

The table below can be used as a guide to show the progression in written calculation throughout the Key Stages although children will go through these stages at varying rates, with the processes often taking place in earlier Year Groups than those suggested in the table.

Reception	Y1	Y2	Y3	Y4	Y5	Y6
Making a recording of a calculation (pictures, symbols, words)						
Jotting to support mental strategy (but dropping jotting when secure in strategy)						
Explaining a mental method						
Developing written methods						

- 3.6 Children should be encouraged to approximate answers using mental methods for every operation before carrying out the formal written calculation. They should be able to explain their estimates.
- 3.7 The written strategies will be used for straightforward algorithms i.e.  $68 + 127 =$ , but they will also be used when working with different units, fractions, decimals, percentages and word problems.

#### 4 Problem Solving

- 4.1 There is a growing emphasis for children to apply their knowledge to solve mathematical problems.
- 4.2 The formal written methods for the four operations should be taught before children are presented with problems that requires their application in a real life context. Problem solving should be introduced as part of the sequence of teaching the formal written method.
- 4.3 Each new method should be practised within a real life context including working with money and measure where appropriate.

#### 5 Planning

- 5.1 This policy is designed to aid medium and short term planning and to be used as a working document to ensure consistency throughout the school.
- 5.2 Planning should be carried out with reference to the current Mathematics Policy, the National Numeracy Strategy Renewed Framework and the Formal Written Calculations Policy. The Mathematics Leader will support all staff with their planning and understanding of this policy.

## 6 Written Calculations

### 6.1 Addition

All calculations should be presented horizontally to enable children the opportunity to calculate mentally first, before progressing onto a formal written method.


#### Step 1

$$3 + 4 = 7 \quad \Leftrightarrow \quad 30 + 40 = 70 \quad \Leftrightarrow \quad 0.3 + 0.4 = 0.7$$

(Links between units, tens and decimals will be made at a later stage)

 Explain and model commutative law:


$$3 + 4 = 7 \quad \Leftrightarrow \quad 4 + 3 = 7$$

 Know all number bonds from numbers 0 – 9, 10, 100 and 1000 and later apply to number bonds to 1.

$$5 + 2 = 7 \quad \Leftrightarrow \quad 7 + 3 = 10 \quad \Leftrightarrow \quad 70 + 30 = 100$$

$$\Leftrightarrow \quad 700 + 300 = 1000 \quad \Leftrightarrow \quad 0.7 + 0.3 = 1.0$$

#### Step 2 -Partitioning


 When adding numbers of two or more digits, model partitioning using **arrow cards**.

$$\begin{array}{r} 36 \\ \swarrow \searrow \\ 30 \quad 6 \end{array} + \begin{array}{r} 24 \\ \swarrow \searrow \\ 20 \quad 4 \end{array} = 60$$

(Answer is transferred back into original calculation)

$$30 + 6 + 20 + 4 = 30 + 20 + 6 + 4 \quad (\text{Arrange putting largest number first})$$

$$\begin{array}{r} 50 \\ \swarrow \searrow \\ 50 \quad 10 \end{array} = 60$$

 The following example shows an alternative method of recording.

$$\begin{array}{r} 36 \\ \swarrow \searrow \\ 30 \quad 6 \end{array} + \begin{array}{r} 24 \\ \swarrow \searrow \\ 20 \quad 4 \end{array} = 60$$

(Answer is transferred back into original calculation)


$$30 + 20 = 50 \quad (\text{Adding the most significant digit first})$$

$$6 + 4 = 10$$

$$50 + 10 = 60$$

## Addition contd


**Step 3 – Expanded vertical method adding most significant numbers first**

 **This is an essential step and must not be missed out.**

 Model this method alongside partitioning to show links.

$$\begin{array}{r}
 163 \quad + \quad 235 \quad = \\
 \swarrow \quad \downarrow \quad \searrow \quad \swarrow \quad \downarrow \quad \searrow \\
 100 + 60 + 3 \quad + \quad 200 + 30 + 5
 \end{array}$$
  

		163		
		+ 235		(No crossing of 10 or 100 boundaries)
		300		
200 + 100 = 300	⇒			
		90		
60 + 30 = 90	⇒			(Add mentally as 300 + 90 + 8 = 398)
		8		
5 + 3 = 8	⇒			
		398		
300 + 90 + 8 = 398	⇒			


 When choosing numbers for calculation, follow this progression:

Numbers crossing **no** boundaries.  
 Numbers crossing **only 10** boundary.  
 Numbers crossing **only 100** boundary.  
 Numbers crossing **both 10 and 100** boundary.

**Step 4 – Expanded vertical method adding least significant numbers first**

$$\begin{array}{r}
 163 \quad + \quad 235 \quad = \\
 \swarrow \quad \downarrow \quad \searrow \quad \swarrow \quad \downarrow \quad \searrow \\
 100 + 60 + 3 \quad + \quad 200 + 30 + 5
 \end{array}$$
  

		163		
		+ 235		(No crossing of 10, 100 or 1000 boundaries)
		8		
5 + 3 = 8	⇒			
		90		
60 + 30 = 90	⇒			
		300		
200 + 100 = 300	⇒			
		398		
8 + 90 + 300 = 398	⇒			


 When choosing numbers for calculation follow guidelines above.

Addition contd

**Step 5 – Compact vertical method**


$$163 + 235 =$$

$$\begin{array}{r} 163 \quad (\text{No crossing of 10, 100 or 1000 boundaries}) \\ + 235 \\ \hline 398 \end{array}$$

 When choosing numbers for calculation follow guidelines above.


**Step 6 – Compact vertical method with carrying, crossing 10, 100 and 1000 boundaries**


$$159 + 235 =$$

 Carry below line, crossing through once number has been added.


$$\begin{array}{r} 159 \quad (\text{Crossing 10's boundary}) \\ + 235 \\ \hline 394 \end{array}$$

± ⇔ “Carry one ten”

 Verbally refer to carrying in terms of carrying tens, hundreds or thousands, etc.

 If children do not understand the compact method then go back to the expanded method.

**Decimals**

 Follow the steps through when working with decimals, changing language to suit numbers being used.

**Step 2**

$$\begin{array}{r} 3.2 \quad + \quad 2.4 \quad = \quad 5.6 \\ \swarrow \quad \searrow \quad \swarrow \quad \searrow \\ 3.0 + 0.2 + 2.0 + 0.4 \end{array} \quad \leftarrow \quad (\text{Answer is transferred back into original calculation})$$

$$3.0 + 2.0 = 5.0$$

$$0.2 + 0.4 = 0.6$$


$$5.0 + 0.6 = 5.6$$

Addition contd


**Step 6 – Compact vertical method with carrying, crossing hundredths, tenths, 1, 10, 100 and 1000 boundaries.**

$$\begin{array}{r} 15.9 \quad (\text{Crossing 1's boundary}) \\ + 23.5 \\ \hline 39.4 \end{array}$$

± ⇨ “Carry one whole”

 Verbally refer to carrying in terms of carrying ones, tenths, hundredths, thousandths, etc.

### **Money & Measure**

 Follow the steps through when working with money and measure, changing the language to suit the units being used.

#### **Step 2**


$$\begin{array}{r} \text{£}3.20 + 2.40 = \text{£}5.60 \\ \swarrow \searrow \quad \swarrow \searrow \\ \text{£}3.00 + 20\text{p} + \text{£}2.00 + 40\text{p} \end{array} \quad \leftarrow \text{(Answer is transferred back into original calculation)}$$

$$\begin{array}{l} \text{£}3.00 + \text{£}2.00 = \text{£}5.00 \\ 20\text{p} + 40\text{p} = 60\text{p} \\ \text{£}5.00 + 60\text{p} = \text{£}5.60 \end{array}$$

**Step 6– Compact vertical method with carrying, crossing 10p, £1, £10, £100 and £1000 boundaries**

$$\begin{array}{r} \text{£}15.90 \quad (\text{Crossing £1's boundary}) \\ + \text{£}23.50 \\ \hline \text{£}39.40 \end{array}$$


± ⇨ “Carry one pound”

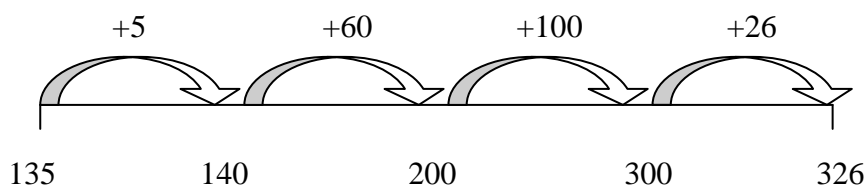
 Verbally refer to carrying in terms of carrying pounds, ten pounds, hundred pounds, etc.

## 4.2 Subtraction


### Step 1 – Complementary addition (Counting on)

$$326 - 135 =$$


 Model the use of the blank number line to lead into the expanded vertical method.



$$100 + 60 + 26 + 5 = 191 \quad (\text{Add most significant number first. Possibility of further partitioning of the 26})$$


 Show jumps above the number line for adding on.

 Size of jumps depends on the ability of the individual child.

 Once confident with the number line method they can begin to use more compact vertical method without the use of a number line.


$$\begin{array}{r}
 326 \\
 - 135 \\
 \hline
 5 \quad (\text{Up to } 140) \\
 60 \quad (\text{Up to } 200) \\
 100 \quad (\text{Up to } 300) \\
 26 \quad (\text{Up to } 326) \\
 \hline
 191
 \end{array}
 \quad (\text{Children write the jumps they have made})$$

$$\begin{array}{r}
 326 \\
 - 135 \\
 \hline
 65 \quad (\text{Up to } 200) \\
 126 \quad (\text{Up to } 326) \\
 \hline
 191
 \end{array}
 \quad \Leftrightarrow \quad
 \begin{array}{r}
 326 \\
 - 135 \\
 \hline
 65 \quad (200) \\
 126 \quad (326) \\
 \hline
 191
 \end{array}$$

 As children consistently show effective use of subtraction using complimentary addition, they can then be taught subtraction by decomposition as a more efficient method.

Subtraction contd

**Step 2 – Expanded decomposition with exchange using partitioning**

 Always refer to the process as ‘exchange’ **rather than** ‘borrowing’.


$$326 - 135 = 191$$

$$\begin{array}{r} 326 \\ - 135 \\ \hline \end{array}$$

(Answer is transferred back into original calculation)

$$\begin{array}{r} 200 \quad 120 \\ 300 + 20 + 6 \\ - 100 + 30 + 5 \\ \hline \end{array}$$

$$100 + 90 + 1 = 191$$

 Use language of 100 **and** 90 **and** 1 instead of 100 add 90 add 1, to avoid confusion with addition within a subtraction calculation.

**Step 3 – Decomposition with exchange**

$$326 - 135 = 191$$

$$\begin{array}{r} 2 \quad 1 \\ 326 \\ - 135 \\ \hline 191 \\ \hline \end{array}$$

 Refer to exchange as “twelve tens” or “two hundreds”, etc.

**Step 4 – Subtraction where answer goes into negative numbers  
(Extension for More Able Pupils)**

$$326 - 135 = 191$$

$$\begin{array}{r} 326 \\ - 135 \\ \hline \end{array}$$

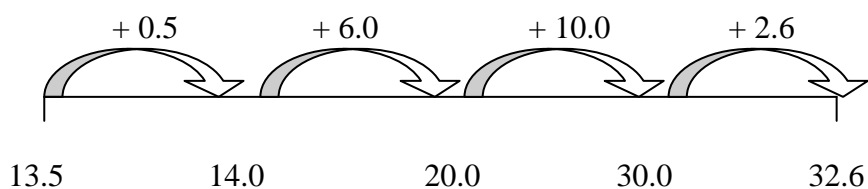
$$\begin{array}{r} 1 \quad (6 - 5 = 1) \\ -10 \quad (20 - 30 = -10) \\ 200 \quad (300 - 100 = 200) \\ \hline 200 - 10 + 1 = 200 - 9 = 191 \end{array}$$

 Ask the children to explain why this method works.

Subtraction contd  
*Decimals*

**Step 1 - Complementary addition (Counting on)**

$$32.6 - 13.5 =$$



$$10.0 + 6.0 + 2.6 + 0.5 = 19.1 \quad (\text{Add most significant number first. Possibility of further partitioning of the 2.6})$$

$$\begin{array}{r}
 32.6 \\
 - 13.5 \\
 \hline
 0.5 \quad (\text{Up to } 14) \\
 6.0 \quad (\text{Up to } 20) \\
 10.0 \quad (\text{Up to } 30) \\
 2.6 \quad (\text{Up to } 32.6) \\
 \hline
 19.1
 \end{array}$$

(Children write the jumps they have made)


**Step 3- Decomposition with exchange**

$$33.8 - 14.9 =$$

$$\begin{array}{r}
 2 \text{ } ^1\text{/}_2 \\
 33 \text{ } ^1\text{/}_8 \\
 - 14.9 \\
 \hline
 18.9
 \end{array}$$

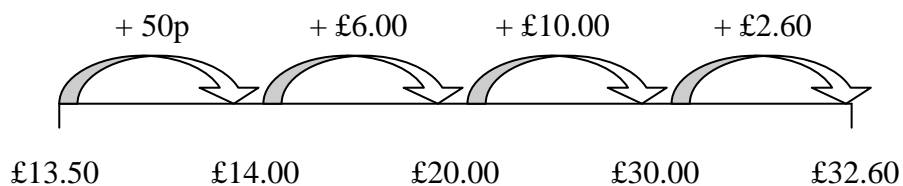
 Refer to exchanges as “eighteen tenths” and “twelve units”, etc.

Subtraction contd  
**Money & Measure**

 Follow the steps through when working with money and measure, changing the language to suit the units being used.


**Step 1 - Complementary addition (Counting on)**


$$£32.60 - £13.50 =$$



$$£10.00 + £6.00 + £2.60 + 50p = £19.10$$


$$\begin{array}{r}
 £32.60 \\
 - £13.50 \\
 \hline
 50p \quad (\text{Up to } £14.00) \\
 £6.00 \quad (\text{Up to } £20.00) \\
 £10.00 \quad (\text{Up to } £30.00) \\
 \hline
 £2.60 \quad (\text{Up to } £32.60) \\
 \hline
 £19.10 \\
 \hline
 \end{array}$$

 Remind children that they need place holders when working with money.

 Make sure that the place value columns are in line.

**Step 3- Decomposition with exchange**

$$\begin{array}{r}
 \phantom{2} \phantom{1} \\
 £32.60 \\
 - £13.50 \\
 \hline
 £19.10 \\
 \hline
 \end{array}$$

 Refer to exchanges as “twelve pounds” and “twenty pounds”, etc.

## 6.3 Multiplication

### Step 1- Partitioning

$$\begin{array}{r}
 17 \times 6 = 102 \\
 \swarrow \searrow \\
 10 + 7
 \end{array}$$

(Refer to + as 'and' not 'add')

$$\begin{array}{r}
 10 \times 6 = 60 \\
 7 \times 6 = 42 \\
 60 + 42 = 102
 \end{array}$$

(Answer is transferred back into original calculation)

### Step 2 – Grid method


$$\begin{array}{r}
 17 \times 6 = 102 \\
 \swarrow \searrow \\
 10 + 7
 \end{array}$$


(Answer is transferred back into original calculation)

x	10	7	
6	60	42	

$$60 + 42 = 102$$

(Answer is transferred back into original calculation)


 When choosing numbers for calculation, initially do not use numbers that cross the 100 boundary. (See ref. 6.1, Step 3, pg 4)


 Extend to HTU x U.

$$\begin{array}{r}
 27 \times 48 = 1296 \\
 \swarrow \searrow \quad \swarrow \searrow \\
 20 + 7 \quad 40 + 8
 \end{array}$$

(Answer is transferred back into original calculation)

x	40	8	
20	800	160	= 960
7	280	56	= 336
			1296

 Children can use any of the methods for addition as shown in section 6.1.

 This method leads on to understanding of algebraic equations in later Key Stages.


Multiplication contd


**Step 3 – Expanded vertical method**

$$\begin{array}{r}
 17 \\
 \times 6 \\
 \hline
 (10 \times 6) \quad 60 \\
 (7 \times 6) \quad 42 \\
 \hline
 102 \\
 \hline
 \end{array}
 \quad \Leftrightarrow \quad
 \begin{array}{r}
 17 \\
 \times 26 \\
 \hline
 (6 \times 7) \quad 42 \\
 (6 \times 10) \quad 60 \\
 (20 \times 7) \quad 140 \\
 (20 \times 10) \quad 200 \\
 \hline
 442 \\
 \hline
 \end{array}$$

**Step 4 – Compact vertical method with carrying**

$$\begin{array}{r}
 17 \\
 \times 6 \\
 \hline
 102 \\
 \hline
 4
 \end{array}$$

 Refer to number carried as “four tens” or “forty”.

 When introducing this method, model the expanded alongside the compact method

**Decimals**

**Step 1- Partitioning**

$$\begin{array}{l}
 1.7 \times 6 = 10.2 \\
 \swarrow \searrow \\
 1.0 + 0.7 \\
 \\
 1.0 \times 6 = 6.0 \\
 \\
 0.7 \times 6 = 4.2 \\
 \\
 6.0 + 4.2 = 10.2
 \end{array}
 \quad \leftarrow \quad
 \begin{array}{l}
 \\
 \\
 \\
 \\
 \\
 \end{array}
 \quad \leftarrow \quad
 \begin{array}{l}
 \\
 \\
 \\
 \\
 \\
 \end{array}$$

(Answer is transferred back into original calculation)



Multiplication contd


**Step 4 – compact vertical method with carrying**

$$\begin{array}{r} \text{£}1.70 \\ \times \quad 6 \\ \hline \text{£}10.20 \\ \hline 4 \end{array}$$


 Refer to number carried as “four pounds”.

## 6.4 Division


**Step 1 – Chunking (Repeated subtraction)**

 Initially use multiples of the divisor.

$$78 \div 6 =$$

 Explain as repeatedly taking out “one lot of 6” and model using practical resource such as jar of cubes.

$$\begin{array}{r} 78 \div 6 \\ - \underline{6} \\ 72 \\ - \underline{6} \\ 66 \\ - \underline{6} \\ 60 \\ - \underline{6} \\ 0 \end{array} \quad \text{(Count lot of 6 until no more lots of 6 can be subtracted)}$$

 Explain answer in terms of “13 lots of 6”.

 Extend to mixed numbers requiring a remainder.





**Step 2 – Chunking (Repeated subtraction using known multiplication facts)**

 Children write key multiplication facts at the side of calculation.



$$\begin{array}{l} 10 \times 6 = 60 \\ 5 \times 6 = 30 \\ 2 \times 6 = 12 \end{array}$$

$\begin{array}{r} 78 \div 6 = 13 \\ - \underline{60} \\ 18 \\ - \underline{12} \\ 6 \\ - \underline{6} \\ 0 \end{array}$	$\leftarrow$	<p>(10 x 6) I have taken <b>10</b> lots of 6</p> <p>(2 x 6) I have taken <b>2</b> lots of 6</p> <p>(1 x 6) I have taken <b>1</b> lot of 6</p> <p><b>10 + 2 + 1 = 13</b> (Add up all of the 'lots of 6')</p>
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Division contd.

-  Use counting stick to model key multiplication facts.
-  Make sure children indicate which numbers need adding at the end during their calculation.
-  The process can be modelled using a number line by counting on or back in multiples of the divisor.
-  This method can be developed to divide HTU by TU with the children taking away larger 'chunks' depending on their knowledge of multiplication.

10 x 36 = 360	$972 \div 36 = 27$	
20 x 36 = 720		
5 x 36 = 180	- 720 (20 x 36)	
2 x 36 = 72	<u>252</u>	
	- 252 (7 x 36)	
	<u>0</u> <b>20 + 7 = 27</b>	

-  Some children may progress to calculating without writing out the multiplication facts.
-  Children will take away different sized 'chunks' according to their ability.


### **Remainders**

10 x 6 = 60	$79 \div 6 = 13 \text{ r } 1$	
5 x 6 = 30		
2 x 6 = 12	- 60 (10 x 6)	
	<u>19</u>	
	- 18 (3 x 6)	
	<u>1</u> <b>10 + 3 = 13 r 1</b>	

### **Decimals**

10 x 3.6 = 36	$97.2 \div 3.6 = 27$	
20 x 3.6 = 72		
5 x 3.6 = 18	- 72.0 (20 x 3.6)	
2 x 3.6 = 7.2	<u>25.2</u>	
	- 25.2 (7 x 3.6)	
	<u>0</u> <b>20 + 7 = 27</b>	

Division contd  
***Money & Measure***

 Remember to first estimate the answer then convert to pence before calculating.

$$£7.80 \div 6 = £1.30$$

$\Leftrightarrow$

$$780\text{p} \div 6 = 130\text{p}$$

$$\begin{array}{r} - 600\text{p} \quad (\mathbf{100\text{p}} \times 6) \\ \hline \end{array}$$

$$\begin{array}{r} 180\text{p} \\ - 120\text{p} \quad (\mathbf{20\text{p}} \times 6) \\ \hline \end{array}$$

$$\begin{array}{r} 60\text{p} \\ - 60\text{p} \quad (\mathbf{10\text{p}} \times 6) \\ \hline \end{array}$$

$$\begin{array}{r} 0 \\ \hline \end{array} \quad \mathbf{100\text{p} + 20\text{p} + 10\text{p} = 130\text{p} = £1.30}$$